ON THE EXISTENCE OF SPECIAL CASES OF INPUT SPEED DOUBLING LINKAGE MECHANISMS

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Abstract  
A special class of planar and spatial linkage mechanisms is presented in which for a continuous full rotation or continuous rocking motion of the input link, the output link undergoes two continuous rocking motions. In a special case of such mechanisms, for periodic motions of the input link with a fundamental frequency $\omega$, the output motion is periodic but with a fundamental frequency of $2\omega$. In this paper, the above class of linkage mechanisms are referred to as speed-doubling linkage mechanisms. Such mechanisms can be cascaded to provide further doubling of the fundamental frequency (rocking motion) of the output motion. They can also be cascaded with other appropriate linkage mechanisms to obtain crank-rocker or crank-crank type of mechanisms. The conditions for the existence of speed-doubling linkage mechanisms are provided and their mode of operation is described in detail. Such speed-doubling mechanisms have practical applications, particularly when higher output speeds are desired, since higher output motions can be achieved with lower input speeds. Such mechanisms also appear to generally have force transmission and dynamics advantages over regular mechanisms designed to achieve similar output speeds.

Keywords: Mechanisms, fundamental frequency, harmonic

1. Introduction

A number of investigators have studied the harmonic content of closed-loop linkage mechanisms with rigid links and have developed different analysis and synthesis methods based on the harmonic content of the output motion [e.g., Freudenstein (1959), Bogdan, et al. (1969), Shaffer and Krause (1962), Midha, et al. (1985), Farhang, et al. (1988), Yuan and Rastegar (2000) and Rastegar and Yuan (2002)]. In general, if the
motion of the input link of a linkage mechanism is periodic with a fundamental frequency $\omega$, the output motion would also be a periodic motion with the same fundamental frequency $\omega$. However, since linkage mechanisms commonly have a nonlinear input-output motion relationship, the output motion would also contain harmonics of the input motion harmonics. For example, if the input link of a four-bar linkage mechanism turns at a constant angular velocity of $\omega$, the output link would undergo a periodic motion with the fundamental frequency $\omega$ and a number of its harmonics. In only a few special cases, e.g., in four-bar parallelogram mechanisms, the input-output relationship is linear and therefore the output motion has the same number of harmonics as the input motion.

The fact that the input and the output motions have to have identical fundamental frequencies can also be explained from the fact that in general, during one cycle of input motion, the output has to complete its motion cycle, therefore should have the same fundamental frequency. For example, in a four-bar crank-crank (crank-rocker) mechanism, one full turn of the input link can only result in one continuous turn (rocking motion) of the output link. In addition, since during one full turn of the input link the coupler and output link chain has to stay within one of their two configurations (see, e.g., Rastegar and Deravi 1987a, 1987b, and Rastegar 1988, 1989 and 1994), the rocker can only make a single continuous back and forth motion between its two extreme positions. This argument is obviously true for any linkage mechanism (Rastegar, 1989).

It can therefore be said that in general, for a continuous full rotation or a continuous rocking motion of the input link of a linkage mechanism, the output link can only undergo a continuous rotation or a continuous rocking motion.

The only exceptions that have been discovered to date are the Galloway\textsuperscript{1} type of mechanisms presented in Rastegar (1990). In these crank-crank type of planar and spatial linkage mechanisms, two turns of the input link results in one full turn of the output link. In Rastegar (1990), it is shown that such motions are possible only in certain special cases. In such cases, one full cycle of the input link rotation occurs in one configuration (branch) and the second cycle in a second configuration of the linkage chain that starts from the output link and extend to the moving joint of the input link. For example, in the Galloway (or deltoid, Hartenberg and Denavit, 1964, or rhomboid, Artobolevsky, 1986) mechanism, during one full rotation of the input link, the open-loop output

\textsuperscript{1}After the English engineer that patented the plane four-bar of it in 1884.
and coupler link chain is in one configuration, and during the second full
turn of the input link, the chain is in its second configuration.

In this paper, a special class of linkage mechanisms is presented in
which for a continuous full rotation or continuous rocking motion of the
input link, the output link undergoes two continuous rocking motions.
In a special case of such mechanisms, for periodic motions of the input
with a fundamental frequency $\omega$, the output motion is periodic but with
a fundamental frequency of $2\omega$. The mechanisms can be cascaded to
provide further doubling of the fundamental frequency (rocking motion)
of the output motion. Such mechanisms may be cascaded with other
appropriate mechanisms to obtain crank-rocker or crank-crank type of
mechanisms. The conditions for the existence of such speed-doubling
linkage mechanisms are provided and their mode of operation is de-
scribed in detail.

Speed-doubling mechanisms have practical applications, particularly
when higher output speeds are desired. This is the case since higher
output motions can be achieved with lower input speeds. Such mech-
anism also appear to generally have force transmission and dynamics
advantages over regular mechanisms designed to achieve similar output
speeds. The latter issues will be discussed in future publications.

2. Input Speed Doubling Linkage Mechanisms

Consider the slider-crank linkage mechanism shown in Fig. 1. The
input link $O_AA$ with the length $a$ makes an angle $\theta$ with the $X$ axis of
the fixed $XY$ coordinate system. The coupler link has a length $b$. The
position of the slider $c$ along the $X$ axis is shown by $s$. If the input link
$O_AA$ undergoes a periodic motion with a fundamental frequency $\omega$, e.g.,
if the motion of the input is the simple harmonic motion

$$\theta = \theta_0 + \theta_1 \sin (\omega t) \quad (1)$$

where $\theta_1$ is the amplitude of the input link oscillation about its position $\theta_0$,
the output motion $s$ will be periodic with the same fundamental fre-
quency $\omega$ and a certain number of its harmonics with significant ampli-
tudes, i.e.,

$$s = s_0 + \sum_{i=1}^{n} s_i \sin(i\omega t + \phi_i) \quad (2)$$

where $n$ is the number of harmonics with significant amplitudes, $s_0$ is a
constant, and $s_i$, $i = 0, 1, \cdots n$ is the constant amplitude and $\phi_i$ is the
phase of the $i$th harmonic of the output motion.

Let a cycle of the aforementioned simple harmonic motion of the input
link start from the position $O_A A$ (solid lines), continue to the position
A′ (dashed lines) during first half of the cycle of motion, and bring the input link back to its starting position \( O_A A \) during the second half of the cycle of motion. During the motion, the output block \( c \) moves from its starting position \( B \) to the position \( B' \) during first half of the cycle of motion, and moves back to the position \( B \) during the second half of the cycle of motion, i.e., during each cycle of motion, the output block \( c \) undergoes one cycle of back and forth motion.

Now, consider the case in which the aforementioned simple harmonic notion of the input link starts from the position \( O_A A \) (solid lines), Fig. 2, continues to the position \( O_A A'' \) (dotted lines), which is symmetrically positioned with respect to the \( X \) axis, during the first half of the cycle of motion, and brings the input link back to its starting position \( O_A A \) during the second half of the cycle of motion. During this motion, the output block \( c \) moves from the position \( B \) to the position \( B' \) as the input link moves from the position \( O_A A \) to the position \( O_A A' \), where the input and the coupler links are collinear, i.e., are in their singular position. As the input link motion continues from the position \( O_A A' \) to the position \( O_A A'' \), the output block \( c \) moves back to its starting position \( B \). The back and forth motion of the output block is repeated as the input link rotates back from its \( O_A A'' \) position to its starting position \( O_A A \). Thus, during one back and forth cycle of the input link motion, the output block \( c \) undergoes two back and forth motions. In this special case of symmetrical motion of the input link about the singular position of the input and coupler links, the two back and forth motions of the output block are identical, each constituting a simple harmonic motion with the fundamental frequency \( 2\omega \). The motion of the output block \( c \), equation (2), is thereby reduced to

\[
s = s_0 + \sum_{i=1}^{m} s_i \sin (2i\omega t + \phi_i)
\]

where \( m \) is the number of harmonics with significant amplitudes.

The output speed is therefore doubled, i.e., the output motion is still harmonic but its fundamental frequency has been doubled. It can also be said that one back and forth motion of the input link results in two back and forth motion of the output block \( c \). The above speed doubling occurs for all input motions as long as the motion during both forward and return half cycles of the input link motion are identical except in their direction.

In the general case of non-symmetrical motion of the input link about its singular (\( \theta = 0 \)) position, the two back and forth motions of the output block are not identical, and the motion of the output block \( c \) is still described by equation (2). However, during one back and forth
cycle of input link motion, the output block \( c \) still undergoes two back and forth motion. The reason for achieving two back and forth motions of the output block \( c \) for each single back and forth motion of the input link is as follows.

The input and coupler chain can place the output block \( c \) in a specified position \( s \) within their reachable space with two different configurations or branches (Rastegar and Deravi 1987a, 1987b, 1989, 1990 and 1994). When the back and forth motion of the input link is with only one of the two configurations of the chain, Fig. 1, the output block can only undergo one back and forth motion, since the functions describing such motions are one to one. Thus, the only way that a single back and forth motion of the input link could result in two back and forth motions of the output block is when one of the latter motions occur in one configuration and the second motion in the other configuration of the input and coupler chain as is shown in Fig. 2. In general, the two back and forth motions of the output block \( c \) are not identical, and together constitute one cycle of a periodic function with the fundamental frequency \( \omega \) of the input motion as described by equation (2). However, when the input motion is symmetrical with respect to the singular position of the input and the couple link chain, the two back and forth motions of the output block become identical, each constituting a simple harmonic motion with the fundamental frequency \( 2\omega \) as described by equation (3).

Similar input speed doubling occurs in all linkage mechanisms when the input link crosses its singular position with the next (coupler like) link during its back and forth (rocking) motion. As the result, the output link undergoes one “back and forth” (rocking) motion in one configuration and a second rocking motion in the other configuration of the input and coupler link chain. For example, such a motion is illustrated in Fig. 3 in a four-bar linkage mechanism. Here, during one cycle of the input link motion, the input link starts its motion from the position \( OA \), pass through the singular position of the input and coupler link \( OA' \) and up to the position \( OA'' \), and continuously returns to its starting position \( OA \).

Similarly, if the two rocking motions of the output link are identical, the fundamental frequency of the output link motion is doubled. The two rocking motions of the output link are identical when the motion of the input link in each of the two configurations of the input and coupler link chain are identical, i.e., the motion from the position \( OA' \) to the position \( OA \) and back is identical to the motion from the position \( OA' \) to the position \( OA'' \) and back, Fig. 3.

In the above two examples, the input link undergoes one rocking motion, crossing the singular position of the input and coupler link chains
during its motion. Such singular position crossings are necessary to allow for one rocking motion of the output in one configuration of the input and coupler link chain and another in the other configuration of the chain. Such a pattern of crossing of the singular position of the input and coupler link chain is obviously not possible if the input link undergoes a full and continuous rotation, i.e., by crank-rocker or crank-crank type of mechanisms. The aforementioned rocking motion of the input link may be, however, generated by another crank-rocker type of mechanism, such as the one shown in Fig. 4. In another scheme, speed-doubling mechanisms may be cascaded to Quadruple the input speed. For example, the output of the mechanism shown in Fig. 4 may be used as an input to a second speed-doubling mechanism as shown in Fig. 5 to further double the input speed to obtained a quadrupled output speed, Fig. 5.

2.1 Example: Input speed doubling in a four-bar linkage mechanism

Consider the four-bar linkage mechanism shown in Fig. 3. Let the link lengths be $a = 3.5 \, \text{cm}$, $b = 6.5 \, \text{cm}$, $c = 7.5 \, \text{cm}$ and $d = 12 \, \text{cm}$. The input motion is considered to be a simple harmonic motion given by

$$\theta = \theta_0 + 30 \cos (\omega t) \quad (4)$$

where $\theta_0$ is the input angle at the singular position of the input and coupler links and $\omega$ is the fundamental frequency of the input motion. With the aforementioned link lengths, the angle $\theta_0$ is readily determined to be $38.52 \, \text{deg}$. Since the input motion, equation (4), is symmetric about the singular position of the input and coupler link chain, the fundamental frequency of the output motion is doubled and the output link undergoes two rocking motions during each cycle of the input motion. For a fundamental frequency of $\omega = 6 \, \text{rad/s}$, the plot of the input and the resulting “speed doubled” output motion are shown in Fig. 6.

3. Discussion and Conclusions

A special class of planar and spatial linkage mechanisms is presented in which for a continuous full rotation or continuous rocking motion of the input link, the output link undergoes two continuous rocking motions. In a special case of such mechanisms, for periodic motions of the input link, the fundamental frequency of the output motion is doubled. In this paper, all the above papers are referred to as “speed-doubling” linkage mechanisms.
The speed-doubling linkage mechanisms can be cascaded to provide further doubling of the fundamental frequency (rocking motion) of the output motion. The mechanisms can also be cascaded with other appropriate linkage mechanisms to obtain crank-rocker or crank-crank type of mechanisms.

The conditions for the existence of speed-doubling linkage mechanisms are provided and their mode of operation is described in detail. Speed-doubling mechanisms have practical applications, particularly when higher output speeds are desired, since higher output motions can be achieved with lower input speeds. Such mechanisms also appear to generally have force transmission and dynamics advantages over regular mechanisms designed to achieve similar output speeds. The latter issues will be discussed in future publications.
References


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Figure 1. A slider-crank linkage mechanism.
Figure 2. A slider-crank linkage mechanism with a symmetric input motion about its singular position $O_A A'$. 
Figure 3. A four-bar linkage mechanism with a symmetric input motion about its singular position $O_A A'$. 
Figure 4. Cascaded two four-bar linkage mechanisms to obtain a speed-doubling crank-rocker linkage mechanism.
Figure 5. The speed-doubling crank-rocker mechanism of Fig. 4 cascaded with a second speed-doubling four-bar linkage mechanism to obtain a crank-rocker mechanism with quadrupled output speed.
Figure 6. Output motion (dashed line) generated by an input motion that is symmetric about the singular position of the input and coupler link chain (solid line) in a four-bar linkage mechanism.