Mechanism Synthesis Theory and the Design of Robots

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Abstract

The synthesis theory for spatial linkage systems can be formulated in a way that is compatible with the geometric design of serial and parallel chain robotic systems. Mechanism design generally seeks exact solutions for a finite approximation to the trajectory of a moving frame. In contrast, robots must have a six-degree-of-freedom workspace with specific characteristics such as shape and size. This paper describe an inventor's environment that fit these workspaces to a designers specification. Current results yield the five-degree-of-freedom TS robot and four-degree-of-freedom CC robot that fit a desired continuous task.

Figure 1: Deployed swing-away and communication board mounts.

1 Introduction

The goal of an inventor is, generally, the specification of a new process or device that satisfies a need. For the "mechanical" inventor the goal is a machine that coordinates movement and applies forces to accomplish a task. While computer-aided design tools are available to generate, analyzed and even manufacture new parts for existing machines, software tools that attempt to identify a new devices matched to a user-specified task are limited to very specific examples.

This paper describes an integrated theory for a computer aided design environment to support the invention of articulated machines for controlled spatial movement. The focus on spatial linkages seeks to bring the broad base of knowledge on the analysis and synthesis of geometric constraints in kinematic synthesis together with the modern insights and capabilities of robotics. While the one-degree-of-freedom planar devices and six-degree-of-freedom spatial devices have been the focus of research for the past decades the broad range of devices between these extremes have received little attention. The challenge of visualizing and analyzing spatial linkages poses a fundamental obstacle to the inventive use of these devices.

2 Design Theory

In contrast to many engineering design situations, the inventor does not have "industrial design concepts" or "product marketing specifications" to guide the design effort. Instead customer surveys identify a generic need and a potential market niche for a here-to-fore unrealized device, such as the wheelchair accessories shown in Figures 1 and 2, see Ruth and McCarthy [27]. Once a technical capability is established in the form of an invention, the commercialization takes place via industrial design and product realization studies, followed by design for manufacturing.

Fundamental to an inventive tool is a functional decomposition of the need expressed by the "voice of the customer," (Ullman [32]). While there are many decomposition strategies, Krishman et al. [13] identify the functions of: motion, power/matter, control and enclosure. For the purposes of articulated systems, this terminology can be focussed by relabeling these functions as: position/orientation workspace, force transmission, coordinated movement/forces, and structural integrity. With this in mind, the task of the invention is defined in terms of the position/orientation workspace of the end-effector, and

Figure 2: Stowed position of the swing-away and communication board mounting linkages.

force transmission, coordination and structural integrity become evaluation metrics.

The basic structure for an inventor's environment can be found in the established practice of engineering design known as *guided iteration*, Dixon and Poli [8]. This refers to formulation of the problem, generation of alternative solutions, evaluation of alternatives, and redesign.

The inventor's environment is essentially a structured idea generation tool. The match between the task specification and the workspace of a candidate class of devices is similar to the search of a catalog, chart or database, Pahl and Beitz [24]; and the evaluation process consists of engineering analysis to determine characteristic measures. Our experience with the development similar design software (Figure 3) has shown that by defining the task via an array of physical positions, or trajectory, of a workpiece and generating a "device map" that categorizes solution classes, we incorporate visual cognition in a scheme of progressive idea generation (Shah [28]). The process informs the inventor about the impact of the task specifications on the availability of candidate solutions, which is particularly important for spatial linkage systems for which model devices do not exist.

3 Linkage Design Theory

The organization of the inventor's environment generalizes existing linkage design practice; see, Suh and Radcliffe [30]. The task of the device is prescribed in terms of a desired workspace and the geometric constraints imposed by the device architecture are resolved to determine its physical dimensions. There are two basic design strategies: the direct solution of

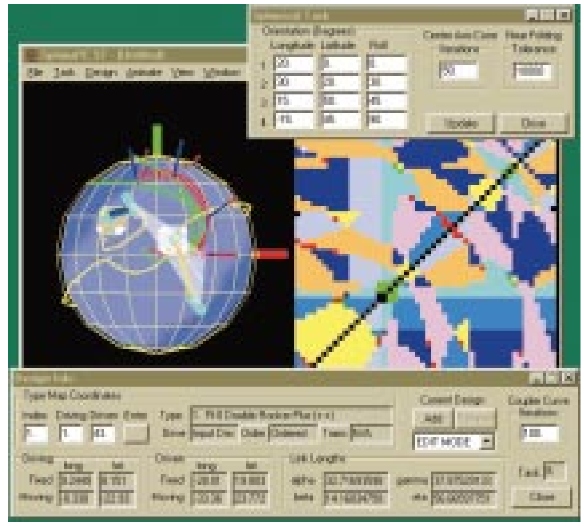

Figure 3: The desk top of `SphinxPC` showing the workspace and device map.

the constraint equations usually with only partial task specification so there is a multi-dimensional set of solutions; and, and optimization strategy that balances compliance with the task specification against performance metrics.

The direct solution of the geometric constraint equations has been applied to the design of planar 4R and 6R linkages, spherical 4R and spatial 4C linkages; see, for example, Erdman and Gustafson [9], Bawab et al. [1], Ruth and McCarthy [26] and Larochelle [15], respectively. In this approach, a discrete task (often four goal positions) is specified that underconstrains the geometric problem so there are many candidate designs. The device candidates are processed to provide advice to the linkage designer and provides the ability to rapidly edit the task and survey candidate designs.

The optimization approach allows a geometrically overconstrained specification of the task as well as the inclusion of performance metrics. See, for example, Starns and Flugrad [29] who compare the "generalized reduced gradient," "genetic algorithm" and "simulated annealing" optimization methods in the design a planar 4R linkage. Camuto and Kinzel [2] present a random walk algorithm that fits a planar 4R to a desired path while also ensuring the device does not jam.

The direct and optimization methods can be combined by considering both the specified task and the workspace reachable by a primitive open chain as manifolds in the "Lie Group" of spatial displacements, often denoted SE(3) (Murray et al. [23]). This approach seeks to fit the "constraint manifold" of a linkage primitive to the desired task manifold. If the task manifold is discrete, then the fit can be made exactly (Murray and McCarthy [21]). If the task manifold is continuous then optimization techniques can be used to fit the constraint manifold to the task manifold (Ravani and Roth [25]). In both cases, the formulation allows for parameterization of candidate designs which yields a device map for the inventor.

4 Task Specification

The concept of a the task manifold is borrowed from robot motion planning. The "configuration space" of a robot is the subset of SE(3) in which it can position its end-effector, often termed its combined reachable and dextrous workspace. Obstacles within the workspace of the robot create forbidden regions in its configuration space. In the free space, key frames are identified and interpolation between these frames define a desired end-effector motion.

For our purposes, the task manifold may be a discrete set of key-frames, a continuous curve representing a specific movement, or multi-dimensional set of positions and orientations encompassing a set of tasks. The goal of the design process is to ensure that the configuration space of the robot encloses the task manifold. This basic idea has been explored in robotics, where the goal is to assemble robotic modules (Chen and Burdick [5]) or select one of a set of candidate robots (Chedmail and Ramstein [3]) to achieve a task that is represented as an approximation to a desired workspace. Murray et al. [22] design a parallel robot by direct evaluation of a constraint manifold primitive evaluated on a task manifold; and Leger and Bares [17] use a genetic algorithm to identify a robot design by evaluating performance metrics on a characteristic task.

A fundamental concern in formulating the design problem is the coordinate frame dependence of representations of SE(3). Any optimization procedure that minimizes a measure of distance between the task manifold and the workspace of a linkage primitive must necessarily depend on the choice of coordinate frames. This means that an optimum design computed in one coordinate frame will, in general, differ from that computed in another frame. There are several strategies for avoiding this situation, see for example, Zefran et al. [33], and Chirikjian [4]. Our experience is that there is a natural bound on the physical size of a task manifold which can be used to identify the error associated with this coordinate-frame dependence (Etzel and McCarthy [10]). In the hands of the inventor this provides insight to the relationship between the task and a successful design (Larochelle and McCarthy [16]).

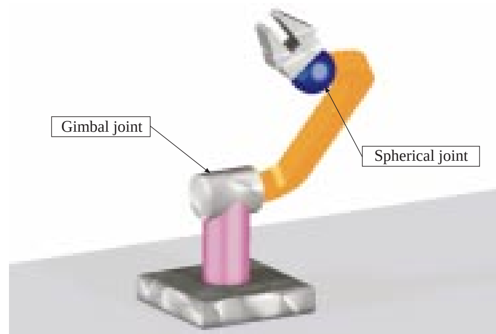

Figure 4: The TS open chain robot.

5 Linkage Primitives

The constraint imposed by a linkage primitive can usually be associated with a geometric object. For example, the TS chain formed by a shoulder joint and a spherical "wrist" (Figure 4) constrains the wrist center to lie on a sphere about the shoulder center. The equations that define these constraints lift to define manifolds in SE(3). Using Clifford Algebra coordinates we obtain algebraic manifolds, called constraint manifolds, that define the position and orientation workspace of the primitive chain (McCarthy [19]).

The constraint manifold of a linkage primitive is a function on the product space SE(3) $\times \mathbf{R}^n$, where $\mathbf{R}^n$ is the space of design parameters. For example, the design parameters of the spatial TS chain are the coordinates of the fixed and moving points $\mathbf{r} = (\mathbf{G}, \mathbf{W}^1)$, and the constraint manifold is the function $\mathcal{L}([T], \mathbf{r})$ on SE(3)$\times\mathbf{R}^6$. For a given set of parameters $\mathbf{r}$, the rotations and translations $[T] \in$ SE(3) that satisfy this equation form the workspace of the chain. This relationship is inverted to determine the set of designs $\mathbf{r} \in \mathbf{R}^n$ that can reach a task manifold specified in SE(3).

While it is known that this inversion can be performed numerically, mechanism synthesis theory shows that direct analytical solutions exist for these primitive chains on discrete tasks. In this case, the discrete tasks can be viewed as control points for manipulating the shape of the constraint manifold. Furlong et al. [11] demonstrate a Virtual Reality interface that enhances the designer's ability to define spatial tasks. This technology may eventually allow direct manipulation of the task and workspace manifolds.

5.1 The spatial TS chain

In this and the following sections, we illustrate the solution procedure that determines the design parameters for a linkage primitive given a discrete task.

The chain constructed from a gimbal joint and a spherical wrist can reach an arbitrary set of seven spatial positions. The design parameter vector is $\mathbf{r} = (x, y, z, \lambda, \mu, \nu)$ is the set of coordinates for both center $\mathbf{G}$ of the gimbal joint and the center of the wrist, $\mathbf{W}^1$. The constraint that characterizes this chain is simply:

$$(\mathbf{W}^i - \mathbf{G}) \cdot (\mathbf{W}^i - \mathbf{G}) = R^2. \quad (1)$$

For a given set of task positions defined by the homogeneous transforms $[T_{1i}], i = 1, \ldots, n$, we have

$$([T_{1i}\mathbf{W}^1 - \mathbf{G}) \cdot ([T_{1i}\mathbf{W}^1 - \mathbf{G}) = R^2, \quad i = 1, \ldots, n. \quad (2)$$

Subtract the first equation from the remaining to obtain:

$$([T_{1i} - I]\mathbf{W}^1) \cdot \mathbf{G} - \frac{1}{2}([T_{1i}]\mathbf{W}^1 \cdot [T_{1i}]\mathbf{W}^1 - \mathbf{W}^1 \cdot \mathbf{W}^1) = 0, \quad i = 2, \ldots, n. \quad (3)$$

Clearly, because there are six unknowns in the design parameter vector, six of these equations, which correspond to seven task positions, completely define the device. An analytical solution exists for these equations that yields as many as 20 TS chains to fit an arbitrary set of seven spatial task positions, see Innocenti [14].

5.2 The spatial CC chain

The CC chain is the generalized robot link that allows both rotation about an axis, and sliding along it (Figure 5). Let $\mathbf{G}$ and $\mathbf{W}^1$ be the directions of the fixed and moving axes, and $\mathbf{P}$ and $\mathbf{Q}^1$ be points on these axes which locate them in space. The CC chain holds the angle and the distance between these axes constant. This yields the geometric constraints:

$$\mathcal{A} : \mathbf{G} \cdot \mathbf{W}^i = |\mathbf{G}||\mathbf{W}^i| \cos\rho,$$
$$\mathcal{M} : (\mathbf{P} \times \mathbf{G}) \cdot \mathbf{W}^i + \mathbf{G} \cdot (\mathbf{Q}^i \times \mathbf{W}^i) = -r|\mathbf{W}^i||\mathbf{G}| \sin\rho \quad (4)$$

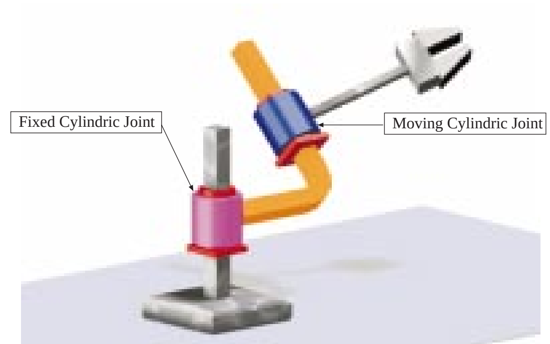

Figure 5: The CC open chain robot.

The design parameter vector for this linkage primitive consists of the components of the Plücker coordinate vectors $\mathsf{G} = (\mathbf{G}, \mathbf{P} \times \mathbf{G})$ and $\mathsf{W}^1 = (\mathbf{W}^1, \mathbf{Q}^1 \times \mathbf{W}^1)$.

For a discrete task specified by n homogeneous transforms $[T_{1i}], i = 1, \ldots, n$, we can construct the set of equations:

$$\mathcal{A}_{1i} : \mathbf{G}^T [A(\phi_{1i}) - I]\mathbf{W}^1 = 0,$$
$$\mathcal{M}_{1i} : \begin{Bmatrix} \mathbf{G} \\ \mathbf{P} \times \mathbf{G} \end{Bmatrix}^T \begin{bmatrix} D_{1i}A_{1i} & A_{1i} - I \\ A_{1i} - I & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{W}^1 \\ \mathbf{Q}^1 \times \mathbf{W}^1 \end{Bmatrix} = 0,$$
$$i = 2, \ldots, n. \quad (5)$$

Mechanism synthesis theory shows that an analytical solution exists that yields as many as six CC chains that reach an arbitrarily specified set of five spatial positions.

5.3 Other primitive chains

There are other chains that may be considered for use as primitives. For example, the spatial CR, RC, and RR chains are each a special case of the CC chain in which a translational freedom along an axis is eliminated. The spatial RR also has an analytical solution (Tsai and Roth [31]). We may also consider robotic style open chains, such as the Cartesian chain PPPS, the PUMA style chain TRS, and leg of a Stewart platform TPS. In the general case, numerical continuation methods can provide all the solutions for a given set of constraint equations (see for example, Dinghra and Zhang [7]). Thus, a modular organization is required that allows the addition of new primitives or combinations of existing primitives.

5.4 Parallel assemblies

The design theory presented above yields multiple solutions for each of the primitive chains. This means

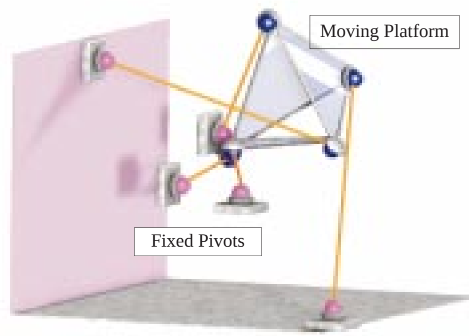

Figure 6: The 5-TS platform linkage.

there are several open chains that can independently reach a prescribed task. This provides the opportunity to connect several chains in parallel configurations. For example, the 20 solutions for a TS chain can be combine in over 15,000 different ways, yielding one degree-of-freedom linkages that reach the same seven spatial positions (Figure 6).

The introduction of parallel chains reduces the number of actuators required for a device but requires the evaluation of singularities of the combined system (Merlet [18]).

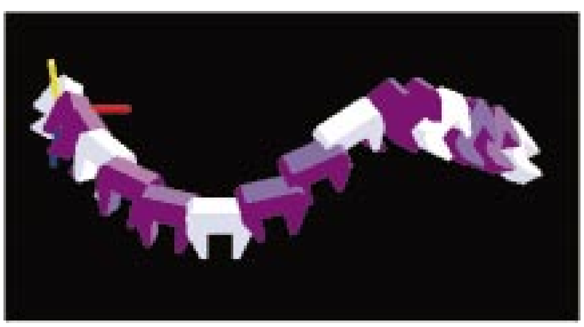

Figure 7: The end-effector trajectory is defined by interpolating key frames.

6 Example Results

Our approach to this Inventor's Environment uses the Bezier interpolation of user specified control frames to define a continuous task trajectory for an end-effector, Figure 7. This is done in the Clifford algebra of 4×4 rotations rather than of 4×4 homogeneous transforms in order to provide coordinate frame invariance for the manifold fitting routines, see McCarthy and Ahlers [20]. Design algorithms for the linkage primitives TS, CC, and RR have been incorporated in the system.

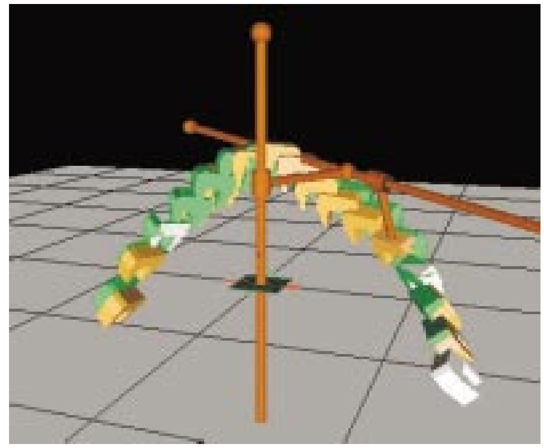

Figure 8: Computer Aided Design for spatial open chains.

Figure 8 illustrates the user defined task and the CC chain that generates the workspace best that fits the task.

7 Conclusion

The design theory for articulated spatial linkages is rich in opportunities for dramatically new devices. An approach exists that integrates classical mechanism synthesis with modern robotic system design. However, software tools that facilitate invention are critical to moving this theory out of the laboratory into industrial practice.

Acknowledgments

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