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DEVELOPING CLASSIFICATIONS FOR SYNTHESIZING, REFINING, AND ANIMATING PLANAR MECHANISMS

David T. Martin

Project Engineer – Ride Dynamics
Delphi Automotive Systems
Kettering, OH 45420-1042
Email: david.t.martin@delphiauto.com

Andrew P. Murray

Assistant Professor
Department of Mechanical and Aerospace Engineering
Dayton, Ohio 45469-0210
Email: murray@udayton.edu

ABSTRACT

In this paper we classify several planar single degree-of-freedom mechanisms for developing features in kinematic synthesis software. The goal of the classification scheme is to identify all mechanisms that behave similarly. Roughly speaking, a mechanism's behavior is assessing whether the input and output links of the mechanisms are capable of a full rotation. The features we use this for are to help identify the range of motion of the input link of a mechanism, to guide the design of a mechanism to result in a desired classification, and to alter the mechanism such that the classification remains unchanged.

INTRODUCTION

Synthesizing and analyzing planar mechanisms via software utilizing interactive graphics is a well-established practice. A myriad of commercially available software allows for the virtual construction and analysis of an unlimited array of mechanisms. There are significantly fewer programs developed to integrate kinematic synthesis into the design process and many of these are for research purposes. Examples of currently available kinematic synthesis software for planar mechanisms include LINCAGES-2000 (www.me.umn.edu/divisions/design/lincages/), Watt (www.heron-technologies.com/software/) and Symech (www.symech.com). A major difference between the software providing virtual construction tools and those performing kinematic synthesis is that the latter suggests (or limits) mechanism topologies (4R, Watt II six-bar, etc.). Beyond planar kinematic synthesis, Larochelle et al. [1] have developed SPHINX for spherical 4R mechanisms and SPADES for spatial 4C mechanisms (Larochelle [2]).

Frequently, the mechanisms generated via kinematic synthesis are classified to find those with desirable properties. A commonly sought property is the existence of a fully rotatable input link. A classification scheme for planar four-bar mechanisms can be found in most machine theory texts including, for example, Erdman and Sandor [3] and Norton [4]. The most generic classification of a four-bar mechanism is as Grashof, having a fully rotatable link, or as non-Grashof. They can then be further classified with the familiar titles of crank-rocker, Grashof double-rocker, drag link, etc. identifying the specific link that is fully rotatable. Proofs of the Grashof criterion can be found in Williams and Reinholtz [5], Paul [6], and the extension to spatial RSSR four-bar linkages in Kazerounian and Solecki [7]. Additional works that seek detailed identifications of the sets of all four-bars are Barker's [8] comprehensive classification of planar four-bar mechanisms and Savage and Hall's [9] similar treatment of spherical four-bars (including discussion by Soni). Spherical 4R mechanisms admit a classification similar to that for planar mechanisms, see Duffy [10] and Chiang [11]. For the spatial RRSS linkage, Su and McCarthy [12] have derived a classification. Building on the idea of the classification of a single mechanism is the classification of all mechanisms of a specific topology solving a single design problem. These "maps" have been included in Sphinx-PC (Ruth and McCarthy [13]) and Lincages-2000 (Chidambaran and Erdman [14]).

Based on the assumption that a designer seeks a specific mechanism topology, we derive classifications for several purposes. The first use is to guide the layout of the mechanism's joints such that a desired classification is achieved. For example, envision the layout of 4R crank-rocker mechanism by selecting, in order, the location of the fixed pivot of the input link, then the input link's moving pivot, then the other moving pivot and finally the remaining fixed pivot. Clearly the four pivots cannot be selected arbitrarily. The second use is to refine a mechanism; to change it such that the classification is left intact. Considering the crank-rocker, where can the moving pivot of the output link be placed (changing two of the link lengths) such that the resulting mechanism remains a crank-rocker? The third use is to expedite the determination of the ranges through which the input link can be driven. SDAMP (Stumph and Murray [15]) implements classification to achieve these three uses. Another use of the classification of note is to determine special cases of mechanisms. The special cases tend to have properties sometimes deemed desirable and other times to be avoided as in the case of the folding four-bar which has one (or more) configuration in which the four pivots are collinear.

The methodology used to determine the mechanism classifications is as follows. Write the loop closure equations for a linkage of a specific topology in terms of the physical parameters to be specified by the design process. Determine the Jacobian with respect to the set of all joint variables. Find the conditions for which the Jacobian loses rank. Use these rank-losing conditions combined with the loop closure equations to develop statements in terms of the physical parameters that, when equal to zero, correspond to the Jacobian losing rank. Evaluate a specific mechanism with these statements. If any of the statements is equal to zero, the mechanism is a special case. Other than zero, only the signs of the statements are important. A second mechanism, evaluated with these statements behaves the same (or is classified the same) if it shares the same signs as the first mechanism. Further, if the signs are different, the mechanism is likely to behave differently. For 4R mechanisms, this methodology defines statement similar to those determined by Bottema and Roth [16] for classification of the image curves of planar four-bar motion.

DERIVING THE 4R MECHANISM CLASSIFICATION

The loop closure equations for the planar 4R mechanism shown in Figure 1 are

$$\begin{aligned} a\cos\Theta_2 + h\cos\Theta_3 - g\cos\Theta_1 - b\cos\Theta_4 &= 0 \\ a\sin\Theta_2 + h\sin\Theta_3 - g\sin\Theta_1 - b\sin\Theta_4 &= 0. \end{aligned} \tag{1}$$

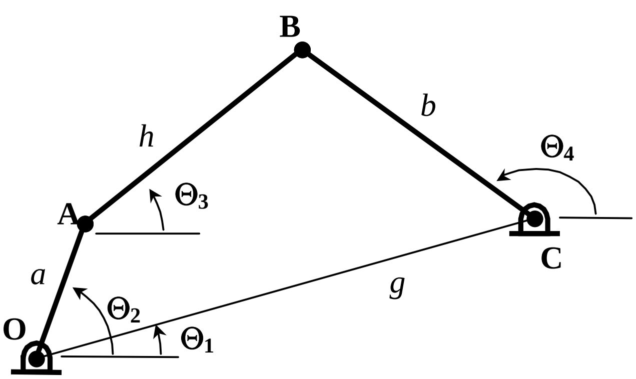

Figure 1. A 4R mechanism's behavior is defined by the four link lengths a, b, g and h.

where a, b, g and h > 0. The Jacobian is

$$\begin{bmatrix} -a\sin\Theta_2 & -h\sin\Theta_3 & b\sin\Theta_4 \\ a\cos\Theta_2 & h\cos\Theta_3 & -b\cos\Theta_4 \end{bmatrix} \begin{Bmatrix} \dot\Theta_2 \\ \dot\Theta_3 \\ \dot\Theta_4 \end{Bmatrix} = 0 \tag{2}$$

In general, the Jacobian's rank is 2. Observe that the conditions to make the Jacobian lose rank are

$$\begin{aligned} \sin(\Theta_2 - \Theta_3) &= 0 \\ \sin(\Theta_2 - \Theta_4) &= 0 \\ \sin(\Theta_3 - \Theta_4) &= 0, \end{aligned} \tag{3}$$

because a, h, g and b cannot be zero. When the three conditions in Eq. 3 are satisfied, the Jacobian is rank 1. There is no way to obtain a rank 0 Jacobian. We now seek the values of the physical parameters, a, h, g, b, and Θ_1 such that the five equations in Eqs. 1 and 3 are simultaneously satisfied.

Each of the conditions in Eq. 3 is satisfied when $\Theta_i - \Theta_j = 0$ or π (i,j = 2,3,4). It would seem that, given two potential solutions for each of the three conditions, there should be a total of eight solutions. However several potential solutions are not possible. For example, if $\Theta_2 - \Theta_3 = \pi$ and $\Theta_2 - \Theta_4 = \pi$, $\Theta_3 - \Theta_4$ must be zero and cannot be π. Thus, Eq. 3 dictates only four possible conditions:

$$\begin{aligned} &\Theta_2 = \Theta_3 = \Theta_4, \\ &\Theta_2 = \Theta_3 = \Theta_4 + \pi, \\ &\Theta_2 = \Theta_3 + \pi = \Theta_4, \\ &\Theta_2 = \Theta_3 + \pi = \Theta_4 + \pi. \end{aligned} \tag{4}$$

Substituting the first relationship, $\Theta_2 = \Theta_3 = \Theta_4$, into Eq. 1 yields

$$\begin{aligned} a\cos\Theta_2 + h\cos\Theta_2 - g\cos\Theta_1 - b\cos\Theta_2 &= 0 \\ a\sin\Theta_2 + h\sin\Theta_2 - g\sin\Theta_1 - b\sin\Theta_2 &= 0. \end{aligned} \tag{5}$$

Multiplying the first equation by $\sin\Theta_2$, the second by $\cos\Theta_2$, and subtracting,

$$g\sin(\Theta_2 - \Theta_1) = 0 \tag{6}$$

and we observe that

$$\Theta_1 = \Theta_2 or \Theta_1 = \Theta_2 + \pi. \tag{7}$$

Substituting $\Theta_2 = \Theta_1$ into Eq. 5,

$$\begin{aligned} a\cos\Theta_1 + h\cos\Theta_1 - g\cos\Theta_1 - b\cos\Theta_1 &= 0 \\ a\sin\Theta_1 + h\sin\Theta_1 - g\sin\Theta_1 - b\sin\Theta_1 &= 0. \end{aligned} \tag{8}$$

We see that if

$$a + h - g - b = 0 \tag{9}$$

the mechanism contains a configuration at which the Jacobian loses rank. We now set $\Theta_1 = \Theta_2 + \pi$, the second option in Eq. 7, and substitute into Eq. 5 and find a second equation,

$$a + h + g - b = 0. \tag{10}$$

Again, the mechanism contains a configuration at which the Jacobian loses rank if the condition in Eq. 10 is satisfied.

We continue this procedure by investigating the second condition in Eq. 4, $\Theta_2 = \Theta_3 = \Theta_4 + \pi$, and performing the same analysis. We then continue through the third and fourth conditions until a set of 8 equations is realized:

$$
\begin{aligned}
T1 &= a+h-g-b=0 \\
T2 &= a+h+g-b=0 \\
T3 &= a+h-g+b=0 \\
T4 &= a+h+g+b=0 \\
T5 &= a-h-g-b=0 \\
T6 &= a-h+g-b=0 \\
T7 &= a-h-g+b=0 \\
T8 &= a-h+g+b=0.
\end{aligned}
\qquad (11)
$$

When any of these conditions is true, the Jacobian in Eq. 2 will lose rank for some configuration of the mechanism. This set of equations can now be used to characterize the behavior of the mechanism.

Evaluating the eight equations in Eq. 11 at values of a, b, g, and h corresponding to a given mechanism will produce values for T1, … T8 that, in general, are not zero. The signs of these equations classify the mechanism. Due to the conditions on the physical parameters, T4 is always positive. The values of T2, T3, T5, and T8 provide assembly conditions for the mechanism. That is, T2, T3, and T8 must be positive for the link lengths to define a realizable mechanism and T5 must be negative. Assuming a 4R linkage to be valid, T1, T6, and T7 provide the sought classification. Table 1 shows the mechanism type corresponding to all possible combinations of signs for T1, T6, and T7. This is the classification derived by Murray and Larochelle [17]. We now use this classification to assist in the layout and refine procedures along with the input link range determination for the 4R mechanism.

Table 1. The 4R linkage classification

	Linkage type	T1	T6	T7
1.	Crank-rocker	-	-	-
2.	Rocker-crank	+	+	-
3.	Double-crank	+	-	+
4.	Grashof double-rocker	-	+	+
5.	00 double-rocker	+	+	+
6.	0π double-rocker	-	+	-
7.	π0 double-rocker	+	-	-
8.	ππ double-rocker	-	-	+

UTILIZING THE 4R MECHANISM CLASSIFICATION

In the layout of a 4R mechanism, we proceed by choosing the locations of pivots O, A, B and C in that order. Our goal is to select these locations such that the resulting mechanism matches one of the desired types in Table 1, for example, a crank-rocker. The locations of the fixed pivot O and the moving pivot A, determining the length of link a, are arbitrary. When we select pivot B, however, we must do so such that T1, T6 and T7 can be less than 0. Equation 11 indicates that h > a satisfiies this condition. Figure 2 shows the crank-rocker layout at the point of selecting pivot B. The shading indicates the region where the inequality, h>a, is true.

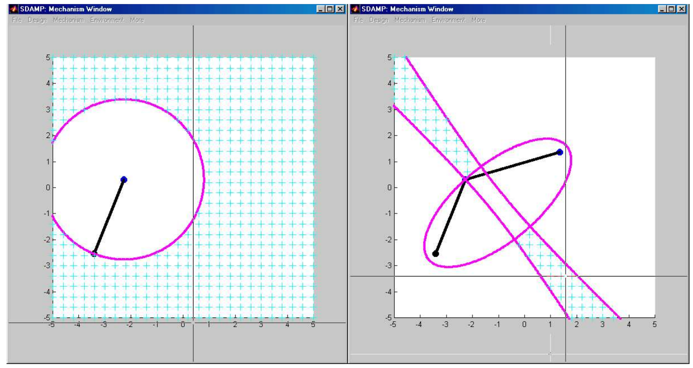

Figure 2. The layout routine for selecting pivot B (at left) and pivot C (at right) that results in a crank-rocker.

The layout concludes with the selection of pivot C, defining the lengths of links b and g. To guarantee that b and g are chosen such that T1, T6, and T7 < 0, the curves T1 = 0, T6 = 0, and T7 = 0 are plotted as shown in Figure 2. Again, the shaded areas indicate the pivot locations that result in the correct signs for T1, T6, and T7.

Now consider the refine procedure, or moving a pivot of the mechanism such that the classification remains unchanged. As shown in Figure 3, we seek to change the location of pivot B, thereby changing the lengths h and b. The curves T1 = 0, T6 = 0, and T7 = 0 (where the lengths h and b are variables) are plotted and the shaded areas indicate where B can be placed without changing the crank-rocker behavior of the mechanism.

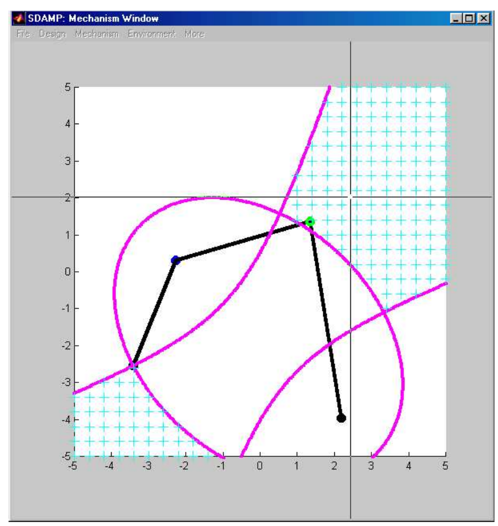

Figure 3. The refine procedure for pivot B for maintaining crank-rocker behavior.

Now consider the input link range determination for a 4R mechanism. The input crank moves in four possible ways:

1. The input link fully rotates;
2. The input link rocks over one range through the angle $\Theta_2=\Theta_1$;
3. The input link rocks over one range through the angle $\Theta_2=\pi+\Theta_1$;

4. The input link rocks through two ranges neither of which contains Θ_1 or $\pi+\Theta_1$.

The classification provides the insight necessary to determine which of these four options is appropriate. More important, it provides a distinction between mechanisms whose input behavior is different. Animation routines can then be constructed based on the input behavior of the mechanism.

DERIVING THE SLIDER-CRANK CLASSIFICATION

The loop closure equations for the slider-crank mechanism shown in Figure 4 are:

$$\begin{aligned} a\cos\Theta_2 + h\cos\Theta_3 - g\sin\Theta_1 - s\cos\Theta_1 - b\cos\Theta_4 &= 0 \\ a\sin\Theta_2 + h\sin\Theta_3 + g\cos\Theta_1 - s\sin\Theta_1 - b\sin\Theta_4 &= 0. \end{aligned} \quad (12)$$

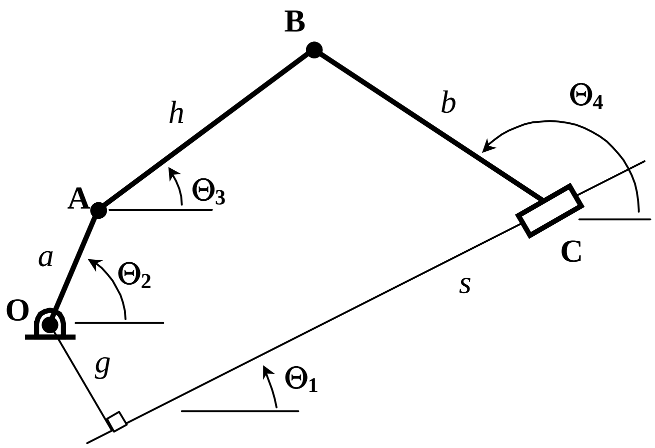

Figure 4. A slider-crank mechanism's behavior is defined by the four link lengths a, b, g, and h along with the angle of the line of slide, Θ_1.

The lengths a, b, h > 0 and g is unrestricted. The Jacobian is

$$\begin{bmatrix} -a\sin\Theta_2 & -h\sin\Theta_3 & -\cos\Theta_1 \\ a\cos\Theta_2 & h\cos\Theta_3 & -\sin\Theta_1 \end{bmatrix} \begin{Bmatrix} \dot\Theta_2 \\ \dot\Theta_3 \\ \dot s \end{Bmatrix} = 0. \quad (13)$$

In general, the rank of the slider-crank's Jacobian is 2. The conditi ons necessary to make the Jacobian lose rank are:

$$\begin{aligned} \sin(\Theta_3 - \Theta_2) &= 0 \\ \cos(\Theta_2 - \Theta_1) &= 0 \\ \cos(\Theta_3 - \Theta_1) &= 0 \end{aligned} \quad (14)$$

When these equations are satisfied, the Jacobian is rank 1. There are no conditions on the joint parameters that will force the Jacobian to become rank 0.

Equation 14 yields four possible combinations of conditions for the joint parameters:

$$\begin{aligned} \Theta_3 = \Theta_2 &= \Theta_1 + \pi/2 \\ \Theta_3 = \Theta_2 + \pi &= \Theta_1 + \pi/2 \\ \Theta_3 + \pi = \Theta_2 &= \Theta_1 + \pi/2 \\ \Theta_3 = \Theta_2 &= \Theta_1 - \pi/2 \end{aligned} \quad (15)$$

Substituting the first relationship, $\Theta_3 = \Theta_2 = \Theta_1+\pi/2$, into the loop closure equations,

$$a + h - g - b\sin(\Theta_4 - \Theta_1) = 0 \quad (16)$$

The mechanism contains a configuration at which the Jacobian will lose rank if Eq. 16 is true. When we substitute the remaining three relationships from eq. 15, one at a time, into the loop closure equations, ultimately we obtain

$$\begin{aligned} T1 &= a + h - g - b\sin(\Theta_4 - \Theta_1) = 0 \\ T2 &= a - h - g - b\sin(\Theta_4 - \Theta_1) = 0 \\ T3 &= a - h + g + b\sin(\Theta_4 - \Theta_1) = 0 \\ T4 &= a + h + g + b\sin(\Theta_4 - \Theta_1) = 0. \end{aligned} \quad (17)$$

Any of the statements in Eq. 17 being true results in the Jacobian losing rank at some configuration. This set of statements provides the classification for the slider-crank mechanism.

Similar to the 4R mechanism, evaluating Eq. 17 at values of a, b, g, h, Θ_1, and Θ_4 corresponding to a given mechanism will produce values for T1, …T4 that, in general, are not zero. The signs of these equations classify the mechanism. Equations T1 and T4 provide assembly conditions for the mechanism. They are positive if the link lengths define a realizable mechanism. Table 2 shows the classification given by the remaining equations, T2 and T3.

Table 2. The slider-crank linkage classification

	Linkage Type	T2	T3
1.	Rocking input with two ranges of motion	+	+
2.	Rocking input with one range of motion (g > 0)	-	+
3.	Rocking input with one range of motion (g < 0)	+	-
4.	Cranking input	-	-

UTILIZING THE SLIDER-CRANK CLASSIFICATION

We now use classifications for the slider crank to assist in the layout and refine procedures and the input link range determination. The layout procedure for a slider-crank mechanism includes the selection of locations for points O, A, B, and C in this order followed by a final selection of a point to indicate the line of slide. In the following example, O, A, B, C, and Θ_4 have been selected such that a fully rotatable input is possible. The selection of a point indicating the line of slide (defining Θ_1), must be made within a specific region. The curves T2=0 and T3=0, in terms of the unknown Θ_1, are plotted and the areas that yield the correct signs for the classifications are shaded (see Figure 5).

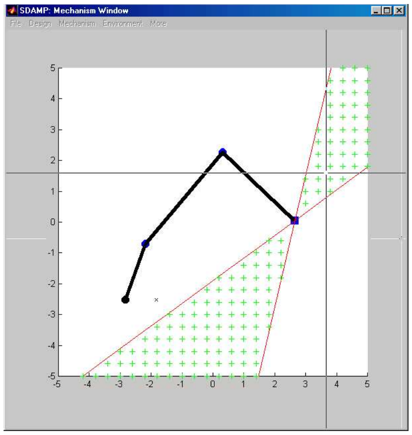

Figure 5. The layout routine for selecting the line of slide to allow a fully rotatable input.

The refine procedure for the slider-crank is handled much the same as the 4R mechanism. Figure 6 shows the curves T2 = 0 and T3 = 0, where the lengths a and g are variable, along with the proper shading for refining pivot O in mechanism with a fully rotatable input.

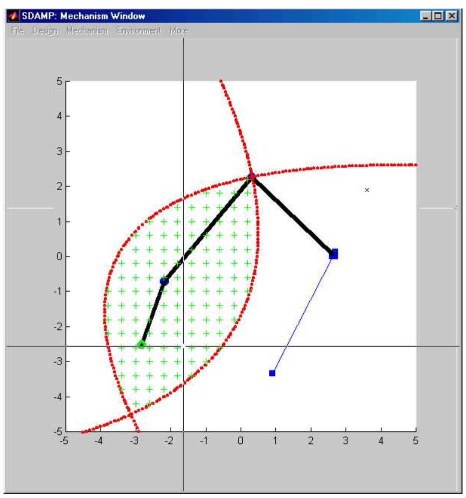

Figure 6. The refine procedure for pivot O to maintain a fully rotatable input.

The input link range determination for the crank-slider must distinguish between the three different behaviors of the input link:

1. The input link fully cranks;
2. The input link rocks through the angle $\Theta_2=\Theta_1-\pi/2$;
3. The input link rocks through the angle $\Theta_2=\Theta_1+\pi/2$;
4. The input link rocks through two ranges, never passing through $\Theta_1-\pi/2$ or $\Theta_1+\pi/2$.

These four input crank motions correspond to the classifications of slider-crank mechanisms shown in Table 2.

INVERTED SLIDER-CRANK CLASSIFICATION

The loop closure equations for the inverted slider-crank shown in Figure 7 are:

$$\begin{aligned} a\cos\Theta_2 + h\cos\Theta_3 - g\cos\Theta_1 - b\cos\Theta_4 &= 0 \\ a\sin\Theta_2 + h\sin\Theta_3 - g\sin\Theta_1 - b\sin\Theta_4 &= 0 \\ \Theta_3 + \gamma - \Theta_4 &= 0. \end{aligned} \qquad (18)$$

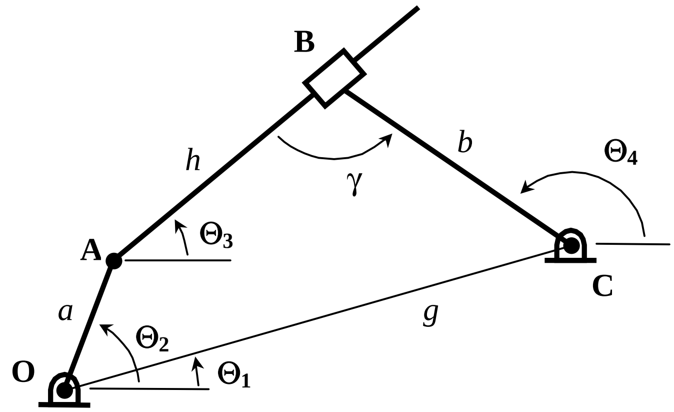

Figure 7. An inverted slider-crank mechanism.

The Jacobian,

$$\begin{bmatrix} -a\sin\Theta_2 & -h\sin\Theta_3 & \cos\Theta_3 & b\sin\Theta_4 \\ a\cos\Theta_2 & h\cos\Theta_3 & \sin\Theta_3 & -b\cos\Theta_4 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{Bmatrix} \dot{\Theta}_2 \\ \dot{\Theta}_3 \\ \dot{h} \\ \dot{\Theta}_4 \end{Bmatrix} = 0, \qquad (19)$$

loses rank under the following conditions

$$\begin{aligned} a\cos(\Theta_2 - \Theta_3) &= 0 \\ h - b\cos\gamma &= 0 \\ ah\sin(\Theta_2 - \Theta_3) + ab\sin(\Theta_4 - \Theta_2) &= 0. \end{aligned} \qquad (20)$$

All three conditions in Eq. 20 are satisfied simultaneously if

$$\begin{aligned} \Theta_3 &= \Theta_2 \pm \pi/2 \\ h &= b\cos\gamma. \end{aligned} \qquad (21)$$

Substituting the conditions from Eq. 21 into the loop closure equations generates

$$\begin{aligned} T1 &= a - g - b\sin\gamma = 0 \\ T2 &= a + g - b\sin\gamma = 0 \\ T3 &= a + g + b\sin\gamma = 0 \\ T4 &= a - g + b\sin\gamma = 0. \end{aligned} \qquad (22)$$

If any of the four above equations are true, the mechanism will contain a configuration at which the Jacobian loses rank. These four equations provide the classification for the inverted slider-crank.

The inverted slider-crank has two classification equations, T2 and T3, that are always positive. These equations provide the assembly conditions for the mechanism.

Table 3. The inverted slider-crank classification

	Linkage Type	T1	T4
1.	Whitworth	+	+
2.	Crank-Shaper	-	-
3.	Rocking input, rocking output ($\gamma < 0$)	+	-
4.	Rocking input, rocking output ($\gamma > 0$)	-	+

CLASSIFICATION OF A WATT II SIX-BAR WITH AN OUTPUT SLIDER

The loop closure equations for the six-bar are

$$\begin{aligned} a\cos\Theta_2 + h\cos\Theta_3 - g_1 - b\cos\Theta_4 &= 0 \\ a\sin\Theta_2 + h\sin\Theta_3 - b\sin\Theta_4 &= 0 \\ d\cos\Theta_5 + e\cos\Theta_6 - g_2 &= 0 \\ d\sin\Theta_5 + e\sin\Theta_6 - s &= 0 \\ \Theta_5 + \gamma - \Theta_4 &= 0 \end{aligned} \tag{23}$$

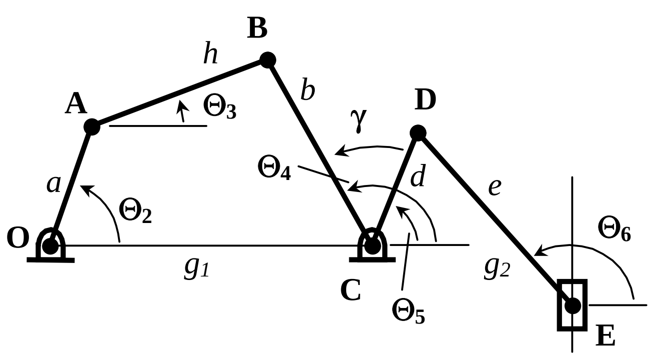

Figure 8. A Watt II six-bar with an output slider.

In order to simplify the analysis, points O and C have been chosen to lie on a horizontal line and a vertical line of slide has been chosen for point E. The Jacobian is

$$\begin{bmatrix} -a\sin\Theta_2 & -h\sin\Theta_3 & b\sin\Theta_4 & 0 & 0 & 0 \\ a\cos\Theta_2 & h\cos\Theta_3 & -b\cos\Theta_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d\sin\Theta_5 & -e\sin\Theta_6 & 0 \\ 0 & 0 & 0 & d\cos\Theta_5 & e\cos\Theta_6 & -1 \\ 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot\Theta_2 \\ \dot\Theta_3 \\ \dot\Theta_4 \\ \dot\Theta_5 \\ \dot\Theta_6 \\ \dot s \end{Bmatrix} = 0. \tag{24}$$

The Jacobian, in general, has a rank of 5. The conditions necessary to obtain a rank of 3, the lowest rank possible for this case, are

$$\begin{aligned} \sin(\Theta_2 - \Theta_3) &= 0 \\ \sin(\Theta_2 - \Theta_4) &= 0 \\ \sin(\Theta_3 - \Theta_4) &= 0 \\ \sin\Theta_5 = \sin\Theta_6 &= 0. \end{aligned} \tag{25}$$

Note that these conditions are simply those of the planar 4R (when Θ_1=0) and slider-crank (when $\Theta_1=\pi$ and b=0). In short, we get all combinations defined by evaluating the 4R and slider-crank separately. As there are eight 4R classifications and four slider-crank classifications, this results in 32 possible classifications of Watt II six-bars with sliding outputs.

The limitation with the classification thus far is shown by examining the case of a crank-rocker connected to a rocking input slider-crank. If the range of motion of the input of the slider-crank encompasses the range of motion of the output of the crank-rocker, then the input of the crank-rocker is fully rotatable. However, if the range of motion of the input of the slider-crank does not encompass the range of motion of the output of the crank-rocker, then the input of the crank-rocker is not fully rotatable.

The previous analysis finds classification equations obtained by forcing the Jacobian to obtain a rank of 3. If the Jacobian becomes rank 4, this is still a loss of rank. This is realized when

$$\begin{aligned} \sin(\Theta_2 - \Theta_3) &= 0 \\ \sin\Theta_6 &= 0. \end{aligned} \tag{26}$$

Combining these conditions with the loop closure equations and solving to find the relationship involving only physical parameters yields

$$\frac{(a\pm h)^2 - g_1^2 - b^2}{2g_1 b} - \frac{g_2 \pm e}{d}\cos\gamma + \frac{\pm\sqrt{d^2 - (g_2 \pm e)^2}}{d}\sin\gamma = 0 \tag{27}$$

If any of these is true, the Jacobian can become rank 4 at some configuration of the six-bar. These terms capture the interaction between the 4R and slider-crank. Implementing these in a useful scheme is the subject of future work.

CONCLUSIONS

This paper shows a methodology useful in classifying several one degree-of-freedom mechanisms. For the 4R mechanism, three conditions resulting in 8 classes of mechanisms are found. As any one of the three terms can go to zero, this results in variety of special cases. For both the slider-crank and the inverted slider-crank, two conditions resulting in four classes of mechanisms are found. The analysis of the Watt II six-bar with a sliding output yields the basic set of classes determined by analyzing the four-bar and slider-crank separately. Furthermore, this methodology yields an additional set of conditions that describe the action of the six-bar due to the interactions between the two mechanisms. The use of classes in guiding the layout of, altering and determining the range of motion for the input length of a mechanism has been emphasized throughout.

REFERENCES

[1] Larochelle, P., Dooley, J., Murray, A., and McCarthy, J., 1993, “SPHINX , Software for Synthesizing Spherical 4R Mechanisms,”

NSF Design and Manufacturing Systems Conference, Charlotte, NC, pp. 607-611.

[2] Larochelle, P., 1998, "SPADES – Software for Synthesizing Spatial 4C Mechanisms," Proceedings, 1998 ASME Design Engineering Technical Conferences, Atlanta, GA.

[3] Erdman, A.G. and Sandor, G.N., 1997, *Advanced Mechanism Design: Analysis and Synthesis*, Vol. 1, Prentice-Hall, Inc., Upper Saddle River, New Jersey.

[4] Norton, R.L., 1992, *Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines*, McGraw-Hill, New York.

[5] Williams, R.L. and Reinholtz, C.F., 1986, "Proof of Grashof's Law Using Polynomial Discriminants," ASME Journal of Mech., Trans., and Auto. in Design, 108, pp. 562-564.

[6] Paul, B., 1979, "A Reassessment of Grashof's Criterion," Journal of Mechanical Design, 101, pp. 515-518.

[7] Kazerounian, K. and Solecki, R., 1993, "Mobility Analysis of General Bi-Modal Four-Bar Linkages Based on Their Transition Angle," Mechanism and Machine Theory, 28, No. 3, pp. 437-445.

[8] Barker, C.R., 1985, "A Complete Classification of Planar Four-Bar Linkages," Mechanism and Machine Theory, 20, No. 6, pp. 535-54.

[9] Savage, M. and Hall, Jr., A.S., 1970, "Unique Descriptions of All Spherical Four-Bar Linkages," Journal of Engineering for Industry, August, pp. 559-566.

[10] Duffy, J., 1980, *Analysis of Mechanisms and Robot Manipulaors*, John Wiley & Sons, Inc., New York.

[11] Chiang, C.H., 1988, *Kinematics of Spherical Mechanisms*, Cambridge University Press, New York.

[12] Su, H. and McCarthy, J.M., 2001, "Classification of Designs for RRSS Linkages," Proceedings, ASME 2001 Design Engineering Technical Conference, Pittsburgh, PA.

[13] Ruth, D.A. and McCarthy J., 1997, "The Design of Spherical 4R Linkages for Four Specified Orientations," Proceedings, ASME Design Technical Conferences, Sacramento, CA.

[14] Chidambaran, C. and Erdman A. G., 1999, "Lincages 2000 Mechanism Design Software.," Proceedings, 1999 Applied Mechanisms and Robotics Conference, Cincinnatti, OH.

[15] Stumph, H.E. and Murray, A.P., 2000, "SDAMP: Software for the Design and Analysis of Mechanical Presses," Proceedings, ASME 2000 Design Engineering Technical Conference, Baltimore, MD.

[16] Bottema, O. and Roth, B., 1979, *Theoretical Kinematics*, North-Holland Publishing Company, New York.

[17] Murray, A. and Larochelle, P., 1998, "A Classification Scheme for Planar 4R, Spherical 4R, and Spatial RCCC Linkages to Facilitate Computer Animation," Proceedings, 1998 ASME Design Engineering Technical Conference, Atlanta, GA.